D = molecular diffusivity, sq.cm./sec.
D_A = molecular diffusivity of A, sq.cm./sec.
L = thickness of a surface element, cm.
N_A = point rate of chemical mass transfer for component A, g.-mole/(sq.cm.)(sec.)
$\overline{N}_A$ = average rate of chemical mass transfer for component A, g.-mole/(sq.cm.)(sec.)
$\overline{N}_A{}^0$ = average rate of physical mass transfer for component A, g.-mole/(sq.cm.)(sec.)
k_L = liquid side mass transfer coefficient with chemical reaction, cm./sec.
k = reaction velocity constant, 1/sec.
n = integer
s = surface renewal rate, 1/sec.
t = time, sec.
u = dummy variable
w = dummy variable
x = distance, cm.

Greek Letters

α = dimensionless group $D_A/(k \cdot L^2)$
β = dimensionless group s/k
γ = dimensionless group, α/β or $D_A/(SL^2)$
ν = exponent

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Comments on Hybrid Computing Time of A DI Method

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A recent article (*1*) by Bishop and Green discusses what appears to be a promising method for hybrid computer solution of multidimensional partial differential equations. However, in making a time comparison between hybrid and digital implementation of the method, the authors make some assumptions which I feel are questionable. They assume that timing will be completely determined by the time it takes the digital to perform its operations. They also assume that hybrid calculations on the digital are carried out using floating point arithmetic.

Because A/D and D/A converters are of limited accuracy, the use of floating point arithmetic is not justified. If the original dimensionless temperature equations are programmed on the analog, one has to A/D convert T^*i, j and D/A convert either the quantity

$$(T^*_{i+1,j} - 2T^*_{i,j} + T^*_{i-1,j}) \tag{1}$$

or

$$(T^*_{i,j+1} - 2T^*_{i,j} + T^*_{i,j-1}) \tag{2}$$

at each mode. This can be accomplished by using single precision fixed points arithmetic and calculating, for example

$$(T^*_{i,j+1} - T^*_{i,j}) + (T^*_{i,j-1} - T^*_{i,j}) \tag{3}$$

The accuracy of the above quantity would be the same as that of the A/D converter. The use of fixed point arithmetic results in the digital portion of the hybrid becoming a very fast function storage and playback device rather than a slow calculating device. For an all digital implementation of the A DI method single precision fixed point arithmetic is out of the question because of the accumulation of numerical errors.

Based on the above considerations I have made a time estimate for digital and hybrid implementation of the A DI method assuming the use of a P DP-8 digital computer. The all digital implementation would take about 6 milliseconds per node assuming .5 msec. per addition and 2 msec. per multiplication or division. For the hybrid implementation the calculation of equation 3 would take about 30 microseconds. Thus, the hybrid appears to be about 200 times faster than the digital. However, the speed of the linkage would be an important factor in the hybrid. Assuming 35 μsec. for an A/D conversion and 10 μsec. for a D/A conversion then the hybrid appears to be about 80 times faster than the digital. It is realized that this estimate is rough, but it is felt that this is a much more realistic estimate than the value of 2 given in the paper. Previous time comparisons which have appeared in the literature (*2* to *4*) have shown that a hybrid computer is anywhere from 20 to 120 times faster than a digital computer for solving partial differential equations.

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